2023

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP - A

1. Answer <u>any eight</u> questions. [1 × 8

(a) Round off the numbers 865250 and 37.46235 to four significant figures.

(b) Multiply the floating point numbers 0.1111E10 and 0.1234E15.

(c) If λ is an eigen value of the matrix A then λ is also an eigen value of A^T. (Is it true ?)

(d) What is Interpolation ?

(e) State the convergence of Newton Raphson method.

(f) What is the Error Term in Lagrangian Interpolation formula ?

(g) If $f(x) = 3x + 1$, $f[x_0, x_1, x_2] = ?$

(h) What is characteristic equation ?

(i) Define central diffeence operator.

(j) State Simpson's $\frac{3}{8}$th rule.

GROUP - B

2. Answer any eight of the following. [1½ × 8

(a) State Newton's Divided Difference formula for Interpolation.

(b) Establish $\Delta = \nabla E$.

(c) Write the error term in Newton's Forward difference Interpolation formula.

(d) State Trapezoidal rule.

(e) Write the interval in which atleast one root of the function $f(x) = x^3 - x - 4$ lies.

(f) Find eigen value of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(g) Use Lagrangian Interpolation formula to find a polynomial satisfying

x	0	1
f(x)	3	4

(h) Prove $E^2f(x_i) = f(x_i + 2h)$, where E is shift operator.

(i) Find the value $\int_1^2 \frac{dx}{x}$ using Trapezoidal Rule, Taking n = 2.

(j) Consider an expression

$$E = \frac{4.2 \times 235.4 \times 0.0246}{7.47 \times 56.3}$$

Find the absolute error in the result.

GROUP - C

3. Answer any eight of the following. [2 × 8

(a) Calculate the relative error in computing $y = x^3 + 3x^2 - x$ for $x = \sqrt{2}$ and $\sqrt{2} = 1.4$.

(b) Compute $\Delta^2\{xf(x)\}$.

(c) Derive $\delta = E^{1/2} - E^{-1/2}$

(d) $\mu^2 = 1 + \frac{1}{4}\delta^2$ (Prove it.)

(e) Find $\frac{dy}{dx}$ for x = 1.0 using

x	1.0	1.1	1.2
f(x)	1.266	1.326	1.393

(f) Locate the real roots of $f(x) = x^3 - 8x + 5$ (only interval).

(g) What is consistent and inconsistent system of equation ?

(h) Compute $\sqrt{2}$ using Newton Raphson Method.

(i) Perform two iterations of Secant method to find the root of $f(x) = x^3 - x + 7$.

(j) Show that, A and A^{-1} have same set of Eigen values.

GROUP - D

4. Answer <u>any four</u> questions.

(a) Evaluate $\int_4^{5.2} \ln x$ using Simpson's $\frac{1}{3}$rd rule and compare the result with exact value taking $h = 0.1$. [6

(b) Find a real root of $\sin x = 10(x - 1)$. [6

(c) Find the eigen value of eigen vector of [6

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

(d) Some by Gauss Jordan method : [6

$$6x + 3y + 2z = 6$$

$$6x + 4y + 3z = 0$$

$$20x + 15y + 12z = 0$$

(e) Derive Lagrangian Interpolation Formula. [6

(f) Using the following table, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for x = 1.0 [6

x	1.0	1.1	1.2	1.3	1.4
f(x)	1.2661	1.3262	1.3937	1.4693	1.5334

(g) Prove that, the Newton-Raphson method is said to have a quadratic rate of convergence. [6

2023

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Full Marks - 80

*Answer **all groups** as per instructions.*
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GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) In a metric space [X, d], d(x, y) = 0, iff ___________.

(b) Every metric is a pseudo metric. (Write True / False)

(c) The metric space defined on real numbers is called – (Choose the most appropriate one.)

(i) Discrete metric space (ii) Indiscrete metric space

(iii) Usual metric space (iv) None of these

(e) R with usual metric is (Choose the most appropriate one.)

(i) 1st countable space (ii) 2nd countable space

(iii) both 1st and 2nd countable spaces

(iv) None of these

(d) The metric space R is a subspace of the metric space C. (Write True / False.)

(f) The space LP, $1 \leq P < \infty$, is separable. (Write True or False.)

(g) If f and g are continuous, then fg is ________.

(h) The image of a Cauchy sequence under a uniform continuous map is Cauchy sequence. (Write True / False.)

(i) The metric space [0, 1] and [0, 2] with the usual metric are homeomorphic. (Write True / False.)

(j) In a metric space every convergent sequence has : (Choose the mosy appropriate one.)

(i) no limit (ii) unique limit

(iii) more than one limit (iv) None of these

(k) The real line is a complete metric space. (Write True / False.)

(l) Every discrete space is ____________. (Choose the most appropriate one.)

(i) Connected (ii) Totally disconnected

(iii) Disconnected (iv) None o f these

GROUP - B

2. Answer any eight of the following. [2 × 8

(a) Define usual metric space.

(b) Define diameter of a set.

(c) What do you mean by limit point of a set ?

(d) Define subspace of a metric space.

(e) Explain 1st countable space.

(f) State Cantor's theorem.

(g) Is R compact ? Explain.

(h) Give an example of a totally bounded metric space.

(i) State contraction mapping principle.

(j) Define nowhere dense subset.

GROUP - C

3. Answer any eight of the following. [3 × 8

(a) Prove that, the union of two open sets is an open set in a metric space.

(b) Prove that a convergent sequence in a metric space is Cauchy sequence.

(c) Prove that in any metric space (X, d), each open ball is an open set.

(d) Prove that, the complement of an open set in a metric space (X, d) is a closed set in X.

(e) Define open and closed sets and give one example of each.

(f) Prove that composition of two uniformly continuous functions is uniformly continuous.

(g) Prove that in any metric space, there is a countable base at each point.

(h) Define interior of a set A, where A is a subset of a metric space (X, d). If X = R and d is the usual metric and A =]1, 2] ∪] 3, 4 [, then write int A.

(i) Is I = [0, 1] homeomorphic to IXI ? Explain.

(j) If a mapping f : X → Y is continuous on X, then prove that $f^{-1}(F)$ is closed in X, for all closed subsets F of Y.

GROUP - D

4. Answer <u>any four</u> questions.

(a) The function d : R × R → R is defined by d(x, y) = |x – y|, ∀ x, y ∈ R. Prove that d is a metric on the set R of all real numbers. [7

(b) Prove that a compact metric space is complete. Is it separable ? Explain. [7

(c) State and prove Baire's category theorem. [7

(d) Let X = R or C be a metric space. If G_1 , G_2 , ... G_n are open subsets of X, then prove that $\bigcap_{i=1}^{n} G_i$ is open. [7

(e) Prove that C [a, b], the set of all continuous functions defined on [a, b] is a complete metric space in the supremum metric. [7

(f) State and prove Cantor's theorem. [7

(g) Let (X, d) be a metric space and Y a subset of X. Then prove that Y is closed and bounded. [7

No. of Printed Pages : 5 **Sem-IV-Math-CC-X(R&B)**

2023

Time - 3 hours

Full Marks - 80

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GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) The set __________ is the trivial subring of R.

(b) The ring of integers is an integral domain. (True/ False)

(c) If $\langle a \rangle = \langle 3 \rangle \langle 4 \rangle$, then what is the value of a ?

(d) Write down the condition under which an integer n with decimal representation $a_k\ a_{k-1}\ ......\ a_0$ is divisible by 9.

(e) Let ϕ is a homomorphism from a ring R to a ring S. ϕ is an isomorphism if and only if ϕ is onto and Ker ϕ = __________.

(f) Every ideal of a ring R is the Kernel of a ring homomorphism of R. (True / False)

(g) Let F be a field, $a \in F$, and $f(x) \in F[x]$. Then what is the remainder in the division of f(x) by $x - a$?

(h) Let F be a field. If $f(x) \in F[x]$ and deg f(x) is 2 or 3, then f(x) is ________ over F if and only if f(x) has a zero in F. (reducible / irreducible)

(i) For any prime p, the pth cyclotomic polynomial

$$\phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \dots\dots + x + 1 \text{ is } \_\_\_\_\_\_\_\_ \text{ over Q.}$$

(reducible / irreducible)

(j) An element a is prime iff ________ is a prime ideal.

(k) Elements a and b of an integral domain D are called associates if a = ________, where u is a unit of D.

(l) Let F be a field. Then F[x] is a unique factorization domain. (True/False)

GROUP - B

2. Answer <u>any eight</u> of the following. [2 × 8

(a) Let a, b and c belong to an integral domian. If $a \neq 0$ and ab = ac, then show that b = c.

(b) If an ideal I of a ring R contains a unit, then show that I = R.

(c) Define integral domain.

(d) Prove that the ideal is not prime in $Z_2[x]$.

(e) Show that the mapping $a + bi \to a - bi$ is a ring homomorphism from $\mathbb{C}$ onto $\mathbb{C}$.

(f) Determine all ring homomorphisms from Z to Z.

(g) Show that $x^2 + 3x + 2$ has four zeros in Z_6.

(h) Show that the polynomial $3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over Q.

(i) Prove that the ring Z is a Euclidean domain.

(j) Define unique factorization domain.

GROUP - C

3. Answer <u>any eight</u> of the following. [3 × 8

(a) Show that the set of Gaussian integers $Z[i] = \{a + bi \mid a, b \in Z\}$ is a subring of the complex number $\mathbb{C}$.

(b) Prove that a ring that is cyclic under addition is commutative.

(c) Show that the ideal $\langle x \rangle$ is a prime ideal in Z[x] but not a maximal ideal in Z[x].

(d) Consider the mapping from $M_2(Z)$ in to Z given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \to a$. Prove or disprove that this is a ring homomorphism.

(e) If D is an integral domain, then show that D[x] is an integral domain.

(f) Let F be a field and let p(x), a(x), b(x) $\in$ F[x]. If p(x) is irreducible over F and p(x) | a(x) . b(x), then show that p(x) | a(x) or p(x) | b(x).

(g) Prove that the polynomial $x^2 - 2$ is irreducible over Q but reducible over $\mathbb{R}$.

(h) Let D be a Euclidean domain with measure d. Show that if a and b are associates in D, then d(a) = d(b).

(i) In Z[i], show that 3 is irreducible but 2 and 5 are not.

(j) Prove that every Euclidean domain is a principal domain.

GROUP - D

4. Answer <u>any four</u> questions.

(a) Prove that a finite integral domain is a field. [7

(b) Prove that the intersection of any set of ideals of a ring is an ideal. [7

(c) Let R be a commutative ring with unity and let A be an ideal of R. Then prove that R/A is an integral domain if and only if A is prime. [7

(d) Let ϕ be a ring homomorphism from a ring R to a ring S. Then prove that Ker $\phi = \{r \in R \mid \phi(r) = 0\}$ is an ideal of R. [7

(e) If F is a field, then show that F[x] is a principal ideal domain. [7

(f) Prove that the product of two primitive polynomials is primitive. [7

(g) Show that the ring $Z[\sqrt{-5}] = \{a + b\sqrt{-5} \mid a, b \in Z\}$ is an integral domain but not a unique factorization domain. [7

2023

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Full Marks - 80

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GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 12

   (a) How many elements are in the power set of the power set of the empty set ?

   (b) Suppose a relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)} on S = {1, 2, 3}. Here R is known as _______ relation.

   (c) Let X = {a, b} and Y = {1, 2, 3}. Then write the number of functions from X to Y.

   (d) If $-1165 \in [r]_7$ then find r ?

   (e) Find the greatest common divisor of 630 and 196.

   (f) If a and b are relatively prime, then HCF (a, b) = ________.

(g) Find the maximum value of $\begin{vmatrix} \sin x & 1 \\ 1 & \cos x \end{vmatrix}$.

(h) If $A = \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix}$ then the cofactor of the element 1 is ________.

(i) Write an example of singular matrix.

(j) A subset 'B' of a vector space V is said to be basis for V is [B] = V. Write the other condition.

(k) What is the position of zero rows if a matrix is said to be in row reduced echelon form ?

(l) Every vector space that is generated by a finite set has basis. Is it true or false ?

GROUP - B

2. Answer any eight of the following. [2 × 8

(a) Prove that ~(~p) = p.

(b) If f(x) = sin x, $g(x) = x^2$, then find gof (x).

(c) If $a \sim b$ defined by $\frac{a}{b}$ is an integer show that ~ is a transitive relation.

(d) Show that if a|c and b|d, then ab|cd, a, b, c, d $\in$ Z.

(e) Solve 5x + 2y = 9 and x – 3y = –5 by using Cramer's rule.

(f) If $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ then find AB.

(g) Show that $\begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}$ is Hermitian matrix.

(h) Prove that the vector (1, 0, 1), (1, 1, 0) and (1, 1, –1) are linearly independent.

(i) In any vector space V, show that (i) OU = 0 (ii) (–1)U = –u for any u $\in$ V.

(j) If T : U $\rightarrow$ V be linear map, then prove that N(T) is subspace of U.

GROUP - C

3. Answer any eight of the following. [3 × 8

(a) Check whether the compound statement $[\sim p \wedge (P \vee q)] \rightarrow q$ is a tautology or not.

(b) Let f : R $\rightarrow$ R defined by $f(x) = \frac{ax}{x+b}$ where a and b are non zero real numbers. Then prove that f is one-to-one.

(c) Show that the open intervals (–2, 2) and (1, 9) have same cardinality.

(d) The relation R in the set of integers given by $R = \{(a, b) : n \mid a - b\}$ is an equivalence relation.

(e) If $a, b \in Z$ and $b \neq 0$, then there exist unique integers q and r such that $a = qb + r$, where $0 \leq r < |b|$.

(f) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$.

(g) Prove that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 2 \end{bmatrix}$ is non-singular and find their inverse.

(h) Prove that the matrix $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ is orthogonal.

(i) In V_2, show that (3, 7) belongs to [(1, 2), (0, 1)].

(j) Prove that the set {(1, 1, 1), (1, –1, 1), (0, 1, 1)} is a basis for V_3.

GROUP - D

4. Answer any four questions.

(a) Show that composition of bijective function is bijective. [7

(b) For a, b ∈ Z define a ~ b if and only if a ≡ b mod 5. [7

(i) Prove that ~ is an equivalence relation.

(ii) Find all equivalence classes for congruence mod 5.

(c) $1.2 + 2.3 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Prove it by method of induction. [7

(d) If $a \equiv x(\bmod n)$ and $b \equiv y(\bmod n)$ then prove that

(i) $a + b \equiv (x + y) \bmod n$

(ii) $ab \equiv xy(\bmod n)$ [7

(e) Determine the rank and nullity of the matrix.

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

by reducing it to row-reduced echelon form. [7

(f) If S is a nonempty subset of a vector space V, then prove that [S] is the smallest subspace of V containing S. [7

(g) Determine the eigen value and the corresponding eigen vectors for the matrix [7

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}.$$